TABLE 11-2. CONVERSION CHART
(Matrices in the same row in the table are equivalent)

$$\Delta_x = x_{11}x_{22} - x_{12}x_{21}$$

	$[z]$	$[y]$	$[T]$	$[T']$	$[h]$	$[g]$
$[z]$	$\begin{matrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{matrix}$	$\begin{matrix} \frac{y_{22}}{\Delta_y} & -\frac{y_{12}}{\Delta_y} \\ -\frac{y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{matrix}$	$\begin{matrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$	$\begin{matrix} \frac{D'}{C'} & \frac{1}{C'} \\ \frac{\Delta_{T'}}{C'} & \frac{A'}{C'} \end{matrix}$	$\begin{matrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{11}} & -\frac{g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\Delta_g}{g_{11}} \end{matrix}$
$[y]$	$\begin{matrix} \frac{z_{22}}{\Delta_z} & -\frac{z_{12}}{\Delta_z} \\ -\frac{z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{matrix}$	$\begin{matrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{matrix}$	$\begin{matrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{A'}{B'} & -\frac{1}{B'} \\ \frac{-\Delta_{T'}}{B'} & \frac{D'}{B'} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta_g}{g_{22}} & \frac{g_{12}}{g_{22}} \\ -\frac{g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{matrix}$
$[T]$	$\begin{matrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{matrix}$	$\begin{matrix} -\frac{y_{22}}{y_{21}} & -\frac{1}{y_{21}} \\ \frac{-\Delta_y}{y_{21}} & -\frac{y_{11}}{y_{21}} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$	$\begin{matrix} \frac{D'}{\Delta_{T'}} & \frac{B'}{\Delta_{T'}} \\ \frac{C'}{\Delta_{T'}} & \frac{A'}{\Delta_{T'}} \end{matrix}$	$\begin{matrix} \frac{-\Delta_h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{21}} & \frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\Delta_g}{g_{21}} \end{matrix}$
$[T']$	$\begin{matrix} \frac{z_{22}}{z_{12}} & \frac{\Delta_z}{z_{12}} \\ \frac{1}{z_{12}} & \frac{z_{11}}{z_{12}} \end{matrix}$	$\begin{matrix} -\frac{y_{11}}{y_{12}} & -\frac{1}{y_{12}} \\ -\frac{\Delta_y}{y_{12}} & -\frac{y_{22}}{y_{12}} \end{matrix}$	$\begin{matrix} \frac{D}{\Delta_T} & \frac{B}{\Delta_T} \\ \frac{C}{\Delta_T} & \frac{A}{\Delta_T} \end{matrix}$	$\begin{matrix} A' & B' \\ C' & D' \end{matrix}$	$\begin{matrix} \frac{1}{h_{12}} & \frac{h_{11}}{h_{12}} \\ \frac{h_{22}}{h_{12}} & \frac{\Delta_h}{h_{12}} \end{matrix}$	$\begin{matrix} -\frac{\Delta_g}{g_{12}} & -\frac{g_{22}}{g_{12}} \\ -\frac{g_{11}}{g_{12}} & -\frac{1}{g_{12}} \end{matrix}$
$[h]$	$\begin{matrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} \frac{B'}{A'} & \frac{1}{A'} \\ \frac{-\Delta_{T'}}{A'} & \frac{C'}{A'} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$	$\begin{matrix} \frac{g_{22}}{\Delta_g} & -\frac{g_{12}}{\Delta_g} \\ -\frac{g_{21}}{\Delta_g} & \frac{g_{11}}{\Delta_g} \end{matrix}$
$[g]$	$\begin{matrix} \frac{1}{z_{11}} & -\frac{z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\Delta_z}{z_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta_y}{y_{22}} & \frac{y_{12}}{y_{22}} \\ -\frac{y_{21}}{y_{22}} & \frac{1}{y_{22}} \end{matrix}$	$\begin{matrix} \frac{C}{A} & -\frac{\Delta_T}{A} \\ \frac{1}{A} & \frac{B}{A} \end{matrix}$	$\begin{matrix} \frac{C'}{D'} & -\frac{1}{D'} \\ \frac{\Delta_{T'}}{D'} & \frac{B'}{D'} \end{matrix}$	$\begin{matrix} \frac{h_{22}}{\Delta_h} & -\frac{h_{12}}{\Delta_h} \\ \frac{-h_{21}}{\Delta_h} & \frac{h_{11}}{\Delta_h} \end{matrix}$	$\begin{matrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{matrix}$

h parameters, and we frequently find it necessary to convert from one set of parameters to another. It is a simple matter to find the relationships of the sets of parameters. For example, comparing Eqs. 11-14 and 11-15 with Eqs. 11-18 and 11-19, we see that

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \frac{1}{\Delta_y}\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix} \tag{11-72}$$

All similar relationships between sets of parameters are summarized in Table 11-2. In this table, the matrices appearing in each of the rows are equivalent. Note that the equivalences involve a factor $\Delta_x = x_{11}x_{22} - x_{12}x$ where x is either z, y, T, T', h or g.

The conditions under which a two-port network is reciprocal are give in Table 11-3 for the six sets of parameters. Also tabulated are the conditior

TABLE 11-3. SOME PARAMETER SIMPLIFICATIONS FOR PASSIVE, RECIPROCAL NETWORKS

Parameter	Condition for ~~Passive~~ Reciprocal Networks	Condition for Electrical Symmetry
z	$z_{12} = z_{21}$	$z_{11} = z_{22}$
y	$y_{12} = y_{21}$	$y_{11} = y_{22}$
$ABCD$	$AD - BC = 1$	$A = D$
$A'B'C'D'$	$A'D' - B'C' = 1$	$A' = D'$
h	$h_{12} = -h_{21}$	$\Delta_h = 1$
g	$g_{12} = -g_{21}$	$\Delta_g = 1$

under which a passive reciprocal two-port network is *symmetrical* in tl sense that the ports may be exchanged without affecting the port voltag and currents.

11-7. Network Functions for Ladders

In this section we show that simple procedures may be followed in cor puting the immittance functions for one special class of network structure the ladder. The ladder network is shown in Fig. 11-12. If each immittan represents one element, the network is known as a *simple* ladder. Otherwis the ladder network may contain *arms* that are arbitrarily complicated, shown by the sample of Fig. 11-13. We follow the practice of characterizi series arms by their impedances and shunt arms by their admittances f reasons that will soon be evident.

We first consider the computation of driving-point immittances for tl ladder network. If we are finding an open-circuit or short-circuit paramet we assume that the appropriate port is prepared by being either open

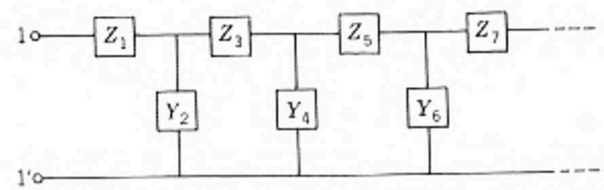


Fig. 11-12. A general ladder network which is described as a simple ladder if each Z or Y describes only one element.

where $[T]$ is the transmission matrix of the over-all network. It is given by

$$(11.7)\quad [T] = [T]_1[T]_2 = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2 \end{bmatrix}$$

It is thus seen that, in the cascade connection of several four-terminal networks, the over-all transmission matrix of the network is the matrix product of the transmission matrices of the individual networks taken in the order of connection. If a four-terminal network is *symmetrical* so that its input and output terminals may be interchanged without altering the current and potential distribution of the network, it may be shown that its transmission matrix has the property that $A = D$.

If the transmission matrices of several fundamental types of electrical circuits are known, then by matrix multiplication it is easy to obtain many useful properties of more complex structures formed by a cascade connection of fundamental circuits. The transmission matrices $[T]$ of several basic electrical circuits are listed in Table 1.

Wave Propagation along a Cascade of Symmetrical Structures. Many important problems of electrical-circuit theory such as those involving

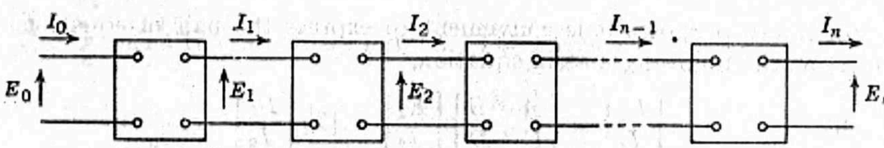


FIG. 11.3. Cascade of four-terminal structures.

electric filters, delay lines, and transducers involve the determination of the nature of the current and potential distribution along a chain of identical symmetric four-terminal networks. Consider the cascade of n identical four-terminal networks as shown in Fig. 11.3. Let each of the four-terminal networks be a symmetrical one with the following transmission matrix:

$$(11.8)\qquad [T] = \begin{bmatrix} A & B \\ C & A \end{bmatrix}$$

Since each of the structures of the chain has the same matrix, the output potential and current E_n and I_n of the nth structure are related to the input potential and current E_0 and I_0 of the first structure by the following equation:

$$(11.9)\qquad \begin{bmatrix} E_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & A \end{bmatrix}^n \begin{bmatrix} E_n \\ I_n \end{bmatrix}$$

In order to obtain a form for the transmission matrix $[T]$ that is convenient for computing powers of $[T]$, introduce the new variables a and

TABLE 1. TRANSMISSION MATRICES OF FUNDAMENTAL FOUR-TERMINAL STRUCTURES

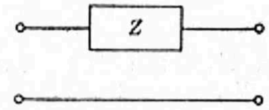
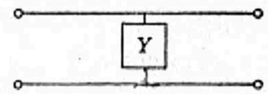
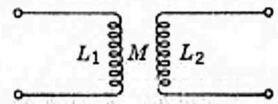
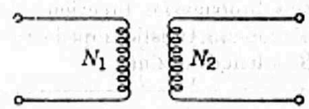
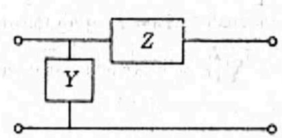
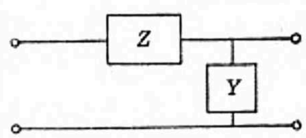
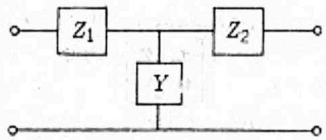
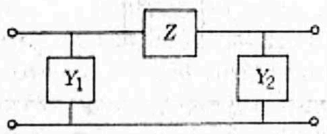
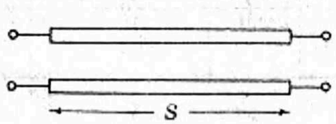
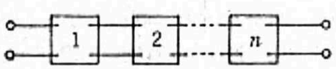
No.	Network	Transmission matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = [T]$
1	 Series impedance	$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$
2	 Shunt admittance	$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$
3	 Coupled circuits	$\begin{bmatrix} \frac{L_1}{M} & \frac{jw(L_1L_2 - M^2)}{M} \\ \frac{-j}{wM} & \frac{L_2}{M} \end{bmatrix}$
4	 Ideal transformer $a = \frac{N_1}{N_2}$	$\begin{bmatrix} \frac{1}{a} & 0 \\ 0 & a \end{bmatrix}$
5		$\begin{bmatrix} 1 & Z \\ Y & 1 + YZ \end{bmatrix}$
6		$\begin{bmatrix} 1 + YZ & Z \\ Y & 1 \end{bmatrix}$

TABLE 1. TRANSMISSION MATRICES OF FUNDAMENTAL FOUR-TERMINAL STRUCTURES (*Continued*)

No.	Network	Transmission matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = [T]$
7	 T section	$\begin{bmatrix} 1 + YZ_1 & Z_1 + Z_2 + YZ_1Z_2 \\ Y & 1 + YZ_2 \end{bmatrix}$
8	 π section	$\begin{bmatrix} 1 + ZY_2 & Z \\ Y_1 + Y_2 + ZY_1Y_2 & 1 + ZY_1 \end{bmatrix}$
9	 Transmission line	$\begin{bmatrix} \cosh aS & Z_0 \sinh aS \\ \dfrac{\sinh aS}{Z_0} & \cosh aS \end{bmatrix}$ where a = propagation function, Z_0 = characteristic impedance, S = length of line
10	 Cascade of symmetrical identical networks. Matrix of individual networks $\begin{bmatrix} A & B \\ C & A \end{bmatrix}$	Over-all matrix of n networks $\begin{bmatrix} \cosh an & Z_0 \sinh an \\ \dfrac{\sinh an}{Z_0} & \cosh an \end{bmatrix}$ where $a = \cosh^{-1} A$ = propagation function, $Z_0 = \sqrt{\dfrac{B}{C}}$ = characteristic impedance

Z_0 by the equations

$$a = \cosh^{-1} A \tag{11.10}$$

and

$$Z_0 = \left(\frac{B}{C}\right)^{\frac{1}{2}} \tag{11.11}$$

With this notation the transmission matrix $[T]$ takes the following form:

$$[T] = \begin{bmatrix} A & B \\ C & A \end{bmatrix} = \begin{bmatrix} \cosh a & Z_0 \sinh a \\ \dfrac{\sinh a}{Z_0} & \cosh a \end{bmatrix} \tag{11.12}$$

If the matrix $[T]$ is multiplied by itself, the following result is obtained:

$$[T][T] = [T]^2 = \begin{bmatrix} \sinh^2 a + \cosh^2 a & Z_0(2 \sinh a \cosh a) \\ \dfrac{2 \sinh a \cosh a}{Z_0} & \sinh^2 a + \cosh^2 a \end{bmatrix} \tag{11.13}$$

$$= \begin{bmatrix} \cosh 2a & Z_0 \sinh 2a \\ \dfrac{\sinh 2a}{Z_0} & \cosh 2a \end{bmatrix}$$

Similarly, by direct multiplication and by the use of the identities of hyperbolic trigonometry, it can be shown that

$$[T]^r = \begin{bmatrix} \cosh ra & Z_0 \sinh ra \\ \dfrac{\sinh ra}{Z_0} & \cosh ra \end{bmatrix} \qquad r = 0, \pm 1, \pm 2, \pm 3, \ldots \tag{11.14}$$

The result (11.14) is very useful in the study of the behavior of four-terminal networks and associated structures. By means of (11.14) Eq. (11.9) may be written in the form

$$\begin{bmatrix} E_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} \cosh an & Z_0 \sinh an \\ \dfrac{\sinh an}{Z_0} & \cosh an \end{bmatrix} \begin{bmatrix} E_n \\ I_n \end{bmatrix} \tag{11.15}$$

or

$$\begin{bmatrix} E_n \\ I_n \end{bmatrix} = \begin{bmatrix} \cosh an & -Z_0 \sinh an \\ -\dfrac{\sinh an}{Z_0} & \cosh an \end{bmatrix} \begin{bmatrix} E_0 \\ I_0 \end{bmatrix} \tag{11.16}$$

The potential E_k and the current I_k along the chain of four-terminal structures of Fig. 11.3 are given by the equation

$$\begin{bmatrix} E_k \\ I_k \end{bmatrix} = \begin{bmatrix} \cosh ak & -Z_0 \sinh ak \\ -\dfrac{\sinh ak}{Z_0} & \cosh ak \end{bmatrix} \begin{bmatrix} E_0 \\ I_0 \end{bmatrix} \tag{11.17}$$

If the chain of four-terminal networks is terminated by an impedance equal to Z_0, then

$$I_n = \frac{E_n}{Z_0} \tag{11.18}$$